

Shear viscosity of a nonperturbative gluon plasma

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Shear viscosity is evaluated within a model of the gluon plasma, which is based entirely on the stochastic nonperturbative fields. We consider two types of excitations of such fields, which are characterized by the thermal correlation lengths $\sim (g^2 T)^{-1}$ and $\sim (g^4 T)^{-1}$, where g is the finite-temperature Yang–Mills coupling. Excitations of the first type correspond to the genuine nonperturbative stochastic Yang–Mills fields, while excitations of the second type mimic the known result for the shear viscosity of the perturbative Yang–Mills plasma. We show that the excitations of the first type produce only an $\mathcal{O}(g^{10})$ -correction to this result. Furthermore, a possible interference between excitations of these two types yields a somewhat larger, $\mathcal{O}(g^7)$, correction to the leading perturbative Yang–Mills result.

Our analysis is based on the Fourier transformed Euclidean Kubo formula, which represents an integral equation for the shear spectral density. This equation is solved by seeking the spectral density in the form of the Lorentzian Ansätze, whose widths are defined by the two thermal correlation lengths and by their mean value, which corresponds to the said interference between the two types of excitations. Thus, within one and the same formalism, we reproduce the known result for the shear viscosity of the perturbative Yang–Mills plasma, and account for possible nonperturbative corrections to it.

I. INTRODUCTION

Over the last ten years, it has been widely discussed that the quark-gluon plasma produced in the RHIC experiments can resemble an almost perfect quantum liquid, which is characterized by the values of the shear-viscosity to the entropy-density ratio, η/s , much smaller than unity [1]. Comparison with the empirical data for water, helium, and nitrogen shows that their (η/s) -ratios reach minima in the vicinity of the corresponding liquid-

gas phase transitions [2]. Given different types of phase transitions and different types of molecules for the above-mentioned three substances, one can guess that such a temperature-behavior of η/s is quite general. Using this observation, one can naturally assume that for the quark-gluon plasma the minimum of the (η/s) -ratio is reached in the vicinity of the deconfinement phase transition. This minimum can be set to the value of $1/(4\pi)$, obtained within $(\mathcal{N} = 4)$ supersymmetric Yang–Mills theory, which was suggested as the lower bound for the (η/s) -ratio [3]. With the increase of temperature, η/s is expected to rise from this bound up to the values predicted by the perturbative-QCD calculations [4]. Thus, a problem can be posed as how to model these essential features in the temperature-behavior of the (η/s) -ratio.

In the present Letter, we address this issue for the purely gluonic plasma, within a model based entirely on the stochastic nonperturbative fields. In particular, we manage to reproduce the said high-temperature behavior of η/s in perturbative Yang–Mills theory by imposing the correlation length of these fields to be $\sim (g^4T)^{-1}$, where g is the finite-temperature Yang–Mills coupling. This correlation length is recognizable as the mean time needed for a parton undergoing Coulomb scatterings in the gluon plasma to deflect by an angle of the order of unity [5]. Of course, besides the ultrasoft momentum scale $\sim g^4T$, stochastic nonperturbative fields possess just the soft scale $\sim g^2T$, which defines the high-temperature behavior of such quantities as the spatial string tension [6] and the nonperturbative gluonic condensate [7]. Hence, the key ingredient of our model is the presence, in the deconfinement phase ($T > T_c$) of interest, of the two types of excitations of the nonperturbative fields. These excitations are characterized by the parametrically different correlation lengths, $\sim (g^2T)^{-1}$ and $\sim (g^4T)^{-1}$. Excitations of the first type describe an extrapolation of the genuine nonperturbative stochastic vacuum Yang–Mills fields to the deconfinement phase [7]. Rather, excitations of the second type are introduced with the purpose to mimic the known perturbative contribution [4] to the shear viscosity.

The goal of the present study is therefore twofold: to quantify the relative contribution to η/s , which is produced by the genuine nonperturbative fields [i.e. those with the correlation length $\sim (g^2T)^{-1}$] with respect to the known perturbative contribution, and to evaluate the contribution to η/s produced by the perturbative-nonperturbative interference. These two issues will be addressed by obtaining the shear spectral density from an integral equation given by the Fourier transformed Kubo formula. That will be done by seeking the spectral

density as a superposition of the Lorentzian Ansätze. As a result, we find that the widths of these Lorentzians are given by the said momentum scales, $\sim g^2 T$ and $\sim g^4 T$, as well as by their mean value (for the interference between the perturbative and nonperturbative interactions).

In the next Section, we perform the corresponding analytic and numerical calculations. In Section III, we give a brief summary of the results obtained.

II. CALCULATION OF THE (η/s) -RATIO

The spectral density $\rho \equiv \rho(\omega, T)$, defining the shear viscosity $\eta \equiv \eta(T)$ as

$$\eta = \pi \left. \frac{d\rho}{d\omega} \right|_{\omega=0}, \quad (1)$$

can be obtained from the following Euclidean Kubo formula [8]:

$$\int_0^\infty d\omega \, \rho \frac{\cosh \left[\omega \left(x_4 - \frac{\beta}{2} \right) \right]}{\sinh(\omega\beta/2)} = \int d^3x \sum_{n=-\infty}^{+\infty} U_T(\mathbf{x}, x_4 + \beta n). \quad (2)$$

Here, $\beta \equiv 1/T$, n labels the winding mode, and U_T is the finite-temperature correlation function of the $(1, 2)$ -component of the Yang–Mills energy-momentum tensor $\Theta_{\mu\nu}$:

$$U_T(\mathbf{x}, x_4 + \beta n) \equiv \langle \Theta_{12}(\mathbf{0}, 0) \Theta_{12}(\mathbf{x}, x_4 + \beta n) \rangle_T, \quad \text{where} \quad \Theta_{12} = g^2 F_{1\mu}^a F_{2\mu}^a. \quad (3)$$

The Kubo formula represents an integral equation for ρ . To solve this equation, we first Fourier transform it. This method of solving the equation is inspired by the observation that

$$\frac{\cosh \left[\omega \left(x_4 - \frac{\beta}{2} \right) \right]}{\sinh(\omega\beta/2)} = 2T \cdot \omega \sum_{k=-\infty}^{+\infty} \frac{e^{i\omega_k x_4}}{\omega^2 + \omega_k^2}, \quad (4)$$

where $\omega_k = 2\pi T k$ is the k -th Matsubara frequency. One further notices that, for non-perturbative fields at issue, U_T exponentially falls off at a distance defined by the thermal correlation length of those fields. For this reason, we consider the maximally general exponential Ansatz for U_T , which is provided by the MacDonald functions. Namely, we start with the following sum, which generalizes the one on the right-hand side of Eq. (4):

$$S \equiv \sum_{k=-\infty}^{+\infty} \frac{e^{i\omega_k x_4}}{(\omega_k^2 + m^2)^\alpha} = \frac{1}{\Gamma(\alpha)} \int_0^\infty d\lambda \, \lambda^{\alpha-1} e^{-\lambda m^2} \sum_{k=-\infty}^{+\infty} e^{-\lambda \omega_k^2 + i\omega_k x_4}.$$

Here, $\Gamma(\alpha)$ stands for the Gamma-function, $\alpha > 0$, and $m = m(T)$ is some mass parameter. The sum over Matsubara modes k can be transformed into a sum over winding modes n , yielding the following intermediate result:

$$S = \frac{\beta}{2\sqrt{\pi}\Gamma(\alpha)} \int_0^\infty d\lambda \lambda^{\alpha-\frac{3}{2}} e^{-\lambda m^2} \sum_{n=-\infty}^{+\infty} e^{-\frac{(x_4+\beta n)^2}{4\lambda}}.$$

One can further multiply this expression by $1 = (4\pi\lambda)^{-3/2} \int d^3x e^{-\frac{\mathbf{x}^2}{4\lambda}}$, which yields

$$S = \frac{\beta}{16\pi^2\Gamma(\alpha)} \int_0^\infty d\lambda \lambda^{\alpha-3} e^{-\lambda m^2} \int d^3x \sum_{n=-\infty}^{+\infty} e^{-\frac{\mathbf{x}^2+(x_4+\beta n)^2}{4\lambda}}.$$

Performing then the λ -integration, one obtains

$$S = \frac{\beta \cdot m^{4-2\alpha}}{2^{\alpha+1}\pi^2\Gamma(\alpha)} \int d^3x \sum_{n=-\infty}^{+\infty} \frac{K_{2-\alpha}(m\sqrt{\mathbf{x}^2+(x_4+\beta n)^2})}{(m\sqrt{\mathbf{x}^2+(x_4+\beta n)^2})^{2-\alpha}},$$

where $K_\nu(x)$ stands for the MacDonald function.

Hence, we assume the correlation function (3) of the following form:

$$U_T(\mathbf{x}, x_4 + \beta n) = N(\alpha) G_T^2 \frac{K_{2-\alpha}(m\sqrt{\mathbf{x}^2+(x_4+\beta n)^2})}{(m\sqrt{\mathbf{x}^2+(x_4+\beta n)^2})^{2-\alpha}}, \quad (5)$$

where $N(\alpha)$ is a numerical parameter, and $G_T \equiv \langle (gF_{\mu\nu}^a)^2 \rangle_T$ is the finite-temperature non-perturbative gluonic condensate. Then the Fourier-transformed Kubo formula reads

$$\int_0^\infty d\omega \frac{\omega \rho}{\omega^2 + \omega_k^2} = \pi^2 2^\alpha N(\alpha) \Gamma(\alpha) G_T^2 \frac{m^{2\alpha-4}}{(\omega_k^2 + m^2)^\alpha}. \quad (6)$$

We use now for ρ a Lorentzian Ansatz with the width m :

$$\rho = \frac{C \omega}{\omega^2 + m^2}, \quad (7)$$

where $C = C(T)$ is the sought function of dimensionality (mass)⁵. Notice that, although the asymptotic freedom requires $\rho \propto \omega^4$ at $\omega \gg T$ (cf. e.g. Ref. [13]), it is the Lorentzian part of ρ which matters for η , since it defines the derivative of ρ at $\omega = 0$. As it then follows from Eq. (1), the shear viscosity is expressed in terms of C as $\eta = \pi C/m^2$. Substituting Ansatz (7) into Eq. (6), we are left with the integral

$$\int_0^\infty d\omega \frac{\omega^2}{(\omega^2 + \omega_k^2)(\omega^2 + m^2)} = \frac{\pi}{2(|\omega_k| + m)}.$$

Setting $\alpha = 1/2$, we arrive at the relation

$$C = \sqrt{8\pi^3} N \frac{G_T^2}{m^3} \frac{|\omega_k| + m}{\sqrt{\omega_k^2 + m^2}},$$

where from now on $N = N(1/2)$. Thus, we see that, for $\alpha = 1/2$, an exponentially falling off function (5) is compatible with the Lorentzian Ansatz (7) for $k = 0$ (that is, in the high-temperature dimensionally-reduced theory) and for $|k| \gg 1$. With a given form (4) of the kernel in the integral equation (2), which is prescribed by the fluctuation-dissipation theorem, and with the use of the Lorentzian Ansatz, a better accuracy can hardly be achieved. Thus, we use the formula

$$\eta \simeq \sqrt{8\pi^5} N \frac{G_T^2}{m^5}, \quad (8)$$

which is supported by the observation that the correcting factor

$$\frac{|\omega_k| + m}{\sqrt{\omega_k^2 + m^2}} \quad (9)$$

is equal to 1 for $T > T_*$, where T_* is the temperature of dimensional reduction. By the end of our analysis, we will numerically evaluate maximum possible deviations of the correcting factor from 1, which can take place for the temperatures $T \in (T_c, T_*)$ at $|k| \sim 1$.

We proceed now to the calculation of the coefficient N . To this end, we first notice that the $(T = 0)$ -counterpart of the function (5) at $\alpha = 1/2$ reads

$$U_0(x) = N G^2 \frac{K_{3/2}(m_0|x|)}{(m_0|x|)^{3/2}}, \quad \text{where} \quad \frac{K_{3/2}(z)}{z^{3/2}} = \sqrt{\frac{\pi}{2}} \frac{1}{z^2} \left(\frac{1}{z} + 1 \right) e^{-z}, \quad (10)$$

$G \equiv \langle (gF_{\mu\nu}^a)^2 \rangle$, and the subscript “0” means “at zero temperature”. Second, we express this correlation function in terms of the 2-point functions of $F_{\mu\nu}^a$ ’s by using the so-called Gaussian-dominance hypothesis. This hypothesis, supported by the lattice simulations [9], states that the connected 4-point function of $F_{\mu\nu}^a$ ’s can be neglected compared to the pairwise products of the 2-point functions. For the function $U_0(x) = \langle g^4 F_{1\mu}^a(0) F_{2\mu}^a(0) F_{1\nu}^b(x) F_{2\nu}^b(x) \rangle$ at issue (cf. Eq. (3)), the Gaussian-dominance hypothesis yields

$$U_0(x) \simeq \langle g^2 F_{1\mu}^a(0) F_{1\nu}^b(x) \rangle \langle g^2 F_{2\mu}^a(0) F_{2\nu}^b(x) \rangle + \langle g^2 F_{1\mu}^a(0) F_{2\nu}^b(x) \rangle \langle g^2 F_{2\mu}^a(0) F_{1\nu}^b(x) \rangle, \quad (11)$$

where we have taken into account that $\langle g^2 F_{1\mu}^a(0) F_{2\mu}^a(0) \rangle = 0$. The contribution of stochastic nonperturbative fields to the function $\langle g^2 F_{\mu\nu}^a(0) F_{\lambda\rho}^b(x) \rangle$ can with a high accuracy be parametrized as [9, 10]

$$\langle g^2 F_{\mu\nu}^a(0) F_{\lambda\rho}^b(x) \rangle = \frac{G}{12} (\delta_{\mu\lambda} \delta_{\nu\rho} - \delta_{\mu\rho} \delta_{\nu\lambda}) \cdot \frac{\delta^{ab}}{N_c^2 - 1} \cdot D(x). \quad (12)$$

In Eq. (12), the dimensionless function $D(x)$ exponentially falls off at a distance called the vacuum correlation length. Substituting Eq. (12) into Eq. (11), one readily obtains $U_0 \simeq \frac{G^2}{72(N_c^2-1)}D^2$. Comparing this expression with Eq. (10), and setting from now on $N_c = 3$, we have

$$D(x) = 24 \left[N \frac{K_{3/2}(m_0|x|)}{(m_0|x|)^{3/2}} \right]^{1/2}. \quad (13)$$

As mentioned above, we consider two types of excitations of the nonperturbative fields, which are characterized by the correlation lengths $\sim (g^2T)^{-1}$ and $\sim (g^4T)^{-1}$. We start with the excitations of the first type. These excitations exist foremost in the confinement phase ($T < T_c$), where they yield the string tension [10] $\sigma = \frac{G}{144} \int d^2x D(\mathbf{x})$ corresponding to the static sources in the fundamental representation. On the other hand, the function $D(x)$ in the confinement phase is conventionally parametrized by just an exponent [9, 10] which, once compared to Eq. (13), would be $D(x) = e^{-m_0|x|/2}$. Plugging both this exponential parametrization and parametrization (13) into the said formula for σ , we obtain the corresponding coefficient N :

$$N = \frac{1}{\left[6 \int_0^\infty dx x^{1/4} \sqrt{K_{3/2}(x)} \right]^2}. \quad (14)$$

From the confinement phase, we immediately jump to the opposite limit of very high temperatures, $T \gg T_c$, where the excitations of the second type are supposed to be mostly important. There, the shear viscosity has the form [4]

$$\eta = \frac{T^3}{g^4} \frac{27.126}{\ln \frac{2.765}{g}}. \quad (15)$$

At such temperatures [7], $G_T \sim (g^2T)^4$, so that Eq. (8) yields $N' \sim \left(\frac{m}{g^4T}\right)^5$. [We use the notation N' to make a distinction from Eq. (14).] In order for this coefficient N' to be constant, one should have $m \sim g^4T$. Thus, the known high-temperature expression for the shear viscosity can indeed be reproduced within a model of nonperturbative stochastic fields with the correlation length $\sim (g^4T)^{-1}$.

To distinguish the two scales, $\sim g^2T$ and $\sim g^4T$, we use from now on the notations M and m , respectively. Thus,

$$M = m_0 f_T \quad \text{and} \quad m = g^2 M. \quad (16)$$

Here, the continuous function f_T can be chosen in the following form:

$$f_T = \begin{cases} \left[\coth\left(\frac{m_0}{4T}\right) \right]^{1/4} & \text{at } T_c < T < T_*, \\ \left[\coth\left(\frac{m_0}{4T_*}\right) \right]^{1/4} \cdot \frac{g^2 T}{g_*^2 T_*} & \text{at } T > T_*. \end{cases} \quad (17)$$

The \coth -factors in Eq. (17) stem from the above-mentioned parametrization $D(x) = e^{-m_0|x|/2}$, which yields [7] $\frac{G_T}{G} = f_T^4 = \coth\left(\frac{m_0}{4T}\right)$ at $T_c < T < T_*$. Furthermore, T_* in Eq. (17) is the temperature of dimensional reduction, and $g_* \equiv g(T_*)$. Lattice simulations [6] and analytic calculations [7, 11] suggest the value of T_* in the range from T_c to $2T_c$. Below, we will find T_* from this range by using the best known lattice value for the shear-viscosity to the entropy-density ratio.

We assume now the function $D(x)$ at $T > T_c$ in the form of a sum

$$D(x) = D_M(x) + D_m(x). \quad (18)$$

Here, D_M is given by Eq. (13) with m_0 replaced by M , while D_m is given by a similar formula: $D_m(x) = 24 \left[N' \frac{K_{3/2}(m|x|)}{(m|x|)^{3/2}} \right]^{1/2}$. Since $M \gg m$ at $T \gg T_c$, one has at such temperatures $D \simeq D_m$ with an exponentially high accuracy. Therefore, the shear viscosity (8) goes at $T \gg T_c$ as

$$\eta \simeq \sqrt{8\pi^5} N' \frac{G_T^2}{m^5} \simeq \sqrt{8\pi^5} N' \frac{G^2}{m_0^5} \cdot \frac{T^3}{g^4} \left\{ \frac{\left[\coth\left(\frac{m_0}{4T_*}\right) \right]^{1/4}}{g_*^2 T_*} \right\}^3.$$

Comparing this expression with the known one, Eq. (15), we get the coefficient N' :

$$N' = \frac{1}{\sqrt{8\pi^5}} \frac{27.126}{\ln \frac{2.765}{g}} \left\{ \frac{g_*^2 T_*}{\left[\coth\left(\frac{m_0}{4T_*}\right) \right]^{1/4}} \right\}^3 \cdot \frac{m_0^5}{G^2}.$$

As it was anticipated above, this result is T -independent with the double logarithmic accuracy (since g depends on T only logarithmically).

We can now proceed towards the main result of the present paper — the full shear viscosity produced by the two types of excitations of the nonperturbative stochastic fields, which are described by the correlation function (18),

$$D(x) = 24 \left\{ \left[N \frac{K_{3/2}(M|x|)}{(M|x|)^{3/2}} \right]^{1/2} + \left[N' \frac{K_{3/2}(m|x|)}{(m|x|)^{3/2}} \right]^{1/2} \right\}.$$

As it follows from the above analysis, the full viscosity is given by the formula

$$\eta = \sqrt{8\pi^5} G_T^2 \left(\frac{N}{M^5} + \frac{N'}{m^5} \right) + \Delta\eta, \quad (19)$$

where the contribution $\Delta\eta$ is produced by the interaction between these two types of excitations. This contribution corresponds to the cross term in the square of the function $D(x)$:

$$\text{cross term} = 24^2 \cdot 2 \left[N N' \frac{K_{3/2}(M|x|)}{(M|x|)^{3/2}} \frac{K_{3/2}(m|x|)}{(m|x|)^{3/2}} \right]^{1/2}. \quad (20)$$

This cross term can be approximated by a function of the type of Eq. (13) as

$$\text{cross term} \simeq 24^2 \cdot \mathcal{N} \frac{K_{3/2}(\mu|x|)}{(\mu|x|)^{3/2}}, \quad (21)$$

where $\mu = \frac{M+m}{2}$. Then, by virtue of Eq. (8), $\Delta\eta$ can be expressed through the mass parameter μ and the yet unknown coefficient \mathcal{N} as

$$\Delta\eta = \sqrt{8\pi^5} \mathcal{N} \frac{G_T^2}{\mu^5}. \quad (22)$$

In terms of the spectral density, this means that the interaction between the two types of excitations is also modeled by the Lorentzian Ansatz (7), whose width μ is given by the mean value of M and m .

Using now the explicit form of the function $\frac{K_{3/2}(z)}{z^{3/2}}$, which can be found in Eq. (10), we see that the approximation (21) to Eq. (20) can be written as follows:

$$\mathcal{N}^2 \simeq \frac{4NN'\mu^4}{M^2m^2} \frac{\left(\frac{1}{M|x|} + 1\right) \left(\frac{1}{m|x|} + 1\right)}{\left(\frac{1}{\mu|x|} + 1\right)^2}.$$

To make the right-hand side of this expression really constant, we have to disregard “+1” in all the three brackets. That is, we restrict ourselves to the leading pre-exponential terms in the exponentially falling off functions $\frac{K_{3/2}(M|x|)}{(M|x|)^{3/2}}$, $\frac{K_{3/2}(m|x|)}{(m|x|)^{3/2}}$, and $\frac{K_{3/2}(\mu|x|)}{(\mu|x|)^{3/2}}$. This yields the following result:

$$\mathcal{N} \simeq 2\sqrt{NN'} \frac{\mu^3}{(Mm)^{3/2}}.$$

Owing to Eq. (22), the sought full shear viscosity (19) finally reads

$$\eta \simeq \sqrt{8\pi^5} G_T^2 \left[\frac{N}{M^5} + \frac{N'}{m^5} + \frac{2\sqrt{NN'}}{\mu^2(Mm)^{3/2}} \right]. \quad (23)$$

At $T \gg T_c$, the three terms on the right-hand side of Eq. (23), once divided by T^3 , scale (with the double logarithmic accuracy) as $\mathcal{O}(g^6)$, $\mathcal{O}(1/g^4)$, and $\mathcal{O}(g^3)$, respectively. We see that, at such temperatures, $\Delta\eta$ is parametrically larger than the contribution of excitations with the correlation length $1/M$ by a factor of $\mathcal{O}(1/g^3)$, while being parametrically smaller than the contribution of excitations with the correlation length $1/m$ by a factor of $\mathcal{O}(g^7)$.

In order to evaluate Eq. (23) numerically, we adopt the lattice-simulated SU(3) Yang–Mills critical temperature $T_c = 270 \text{ MeV}$ and the two-loop running coupling [6]

$$g^{-2} = 2b_0 \ln \frac{T}{\Lambda} + \frac{b_1}{b_0} \ln \left(2 \ln \frac{T}{\Lambda} \right), \text{ where } b_0 = \frac{11N_c}{48\pi^2}, b_1 = \frac{34}{3} \left(\frac{N_c}{16\pi^2} \right)^2, \Lambda = 0.104T_c, N_c = 3.$$

For the correlation length $\frac{m_0}{2}$ of the SU(3) Yang–Mills vacuum in the confinement phase, entering Eq. (16), we use the lattice value from Ref. [9], which reads $\frac{m_0}{2} = 894 \text{ MeV}$. Furthermore, also in the confinement phase, we should choose such a lattice value of the gluon condensate G that corresponds to an Ansatz for the two-point correlation function (12) without the $\frac{1}{|x|^4}$ -term. According to Ref. [12], this value is $G = 2.21 \text{ GeV}^4$. Next, the entropy density, $s = s(T)$, in which units η is measured, can be obtained by the formula $s = dp_{\text{lat}}/dT$, where we use for the pressure p_{lat} the corresponding lattice values from Ref. [6]. The value $\frac{s(T_c)}{T_c^3} \simeq 2.2$ is fixed by imposing cancellation of the latent-heat contribution to the pressure. Namely, this contribution is expressed by the discontinuity of ε/T^4 at $T = T_c$, which, according to Fig. 7 of Ref. [6], is $\simeq 2.2$. In Fig. 1, we plot the resulting s/T^3 as a function of T/T_c .

Implying the subtraction of the vacuum contribution, and noticing that $\coth\left(\frac{m_0}{4T_c}\right) = 1.07 \simeq 1$, we use in Eq. (23) a parametrization of G_T in the form $G_T = G \cdot (f_T^4 - 1)$, where f_T is given by Eq. (17). We find the value of T_* upon the comparison with the best known lattice value of the $\frac{\eta}{s}$ -ratio [13], which reads $\frac{\eta}{s}|_{T=1.65T_c} = 0.134$. Then, the value of T_* comes out to be just $T_* \simeq 1.65 T_c$, and the resulting $\frac{\eta}{s}$ turns out to be a monotonically increasing function with the minimum value at $T = T_c$: $\frac{\eta}{s}|_{T=T_c} \simeq 0.081$. We notice that this value is only a few per cent larger than the lower bound of $\frac{1}{4\pi}$ predicted by the AdS/CFT-correspondence [3]. The resulting $\frac{\eta}{s}$ -ratio is plotted in Fig. 2. Also, using this value of T_* , we plot in Fig. 3 the ratios of the 1st and the 3rd terms on the right-hand side of Eq. (23) to the leading, 2nd, term. As mentioned above, the first of these two ratios goes as $\mathcal{O}(g^{10})$, becoming at $T = 4.5 T_c$ as small as only 0.01, whereas the second ratio goes as $\mathcal{O}(g^7)$.

Furthermore, we evaluate the extent to which the correcting factor (9) at $|k| \sim 1$ may

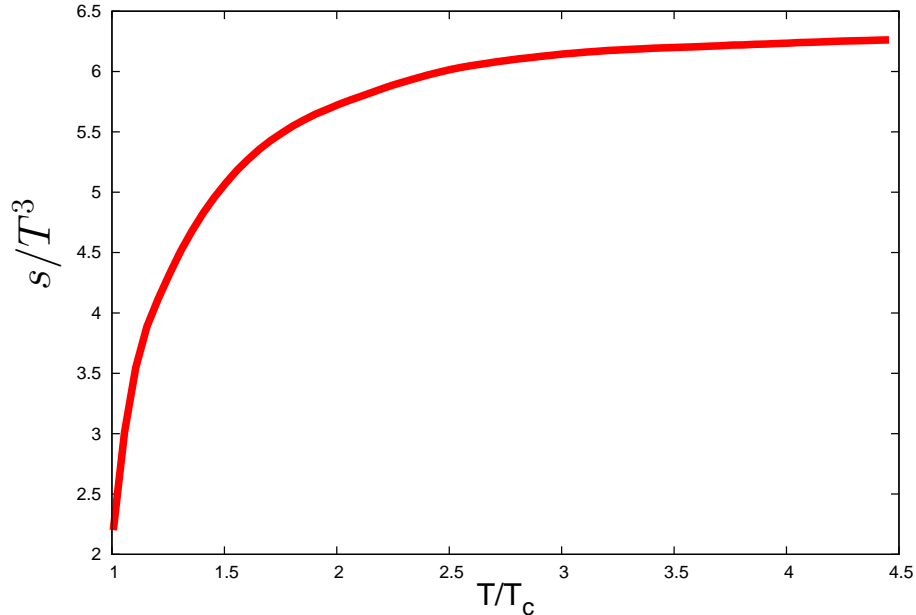


Figure 1: The (s/T^3) -ratio corresponding to the lattice values for the pressure from Ref. [6] (courtesy of F. Karsch), as a function of T/T_c .

differ from 1 for temperatures $T \in (T_c, T_*)$. It turns out that, regardless of the argument, m , M , or μ , entering this correcting factor, the maximum value it can reach is < 1.42 . Consequently, at $T \in (T_c, T_*)$, the resulting $\frac{\eta}{s}$ may be up to 42% larger. That would lead to a discontinuity at $T = T_*$ of the curve depicted in Fig. 2, since in the dimensionally-reduced phase of $T > T_*$ the correcting factor (9) is always equal to 1. This uncertainty, emerging due to possible deviations of the correcting factor from 1 at $T \in (T_c, T_*)$, is related to a nonuniversal character of the Lorentzian Ansatz (7).

III. CONCLUDING REMARKS

In this Letter, we have considered shear viscosity within a model of the gluon plasma based entirely on the nonperturbative stochastic fields which, however, may develop excitations with two different correlation lengths, $\sim (g^2 T)^{-1}$ and $\sim (g^4 T)^{-1}$. Excitations of the first type possess all the features of the genuine nonperturbative stochastic chromo-magnetic fields, which survive the deconfinement phase transition in Yang–Mills theory [9]. Rather, excitations of the second type are introduced for the sole purpose to reproduce, within one and the same model, the known high-temperature perturbative contribution to the shear

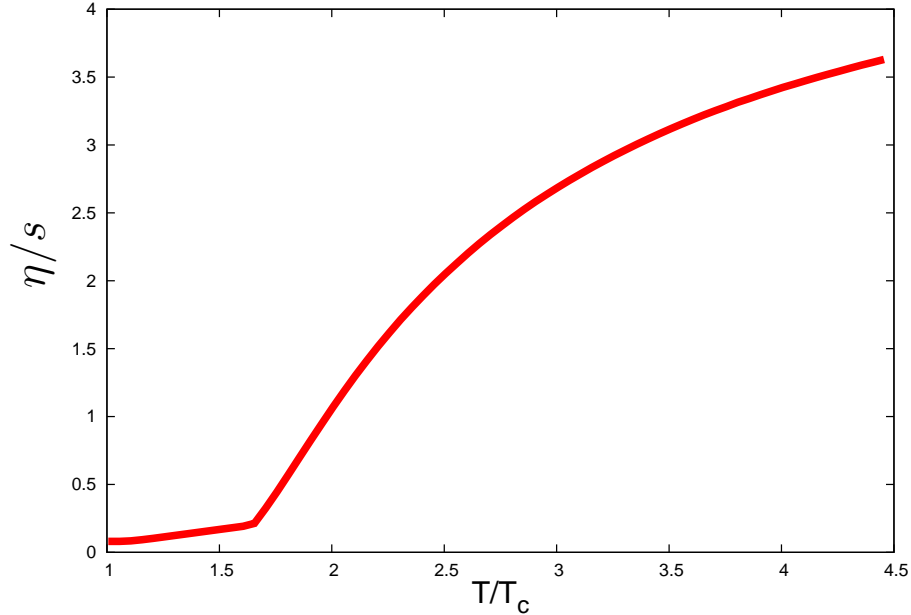


Figure 2: The evaluated (η/s) -ratio as a function of T/T_c .

viscosity. Bringing both types of excitations together, we have studied the contribution of nonperturbative stochastic Yang–Mills fields to the shear viscosity, relative to the perturbative contribution, as well as the result of the perturbative-nonperturbative interference (cf. Fig. 3). Remarkably, the interference contribution parametrically exceeds the purely nonperturbative one by a factor of $\mathcal{O}(1/g^3)$. Furthermore, using the best known lattice value for the $\frac{\eta}{s}$ -ratio from Ref. [13], we have evaluated the temperature dependence of this ratio, which is depicted in Fig. 2. The minimum value of the resulting $\frac{\eta}{s}$ -ratio, reached at $T = T_c$, turns out to be only a few per cent larger than the lower bound of $\frac{1}{4\pi}$ predicted for this quantity by the AdS/CFT-correspondence.

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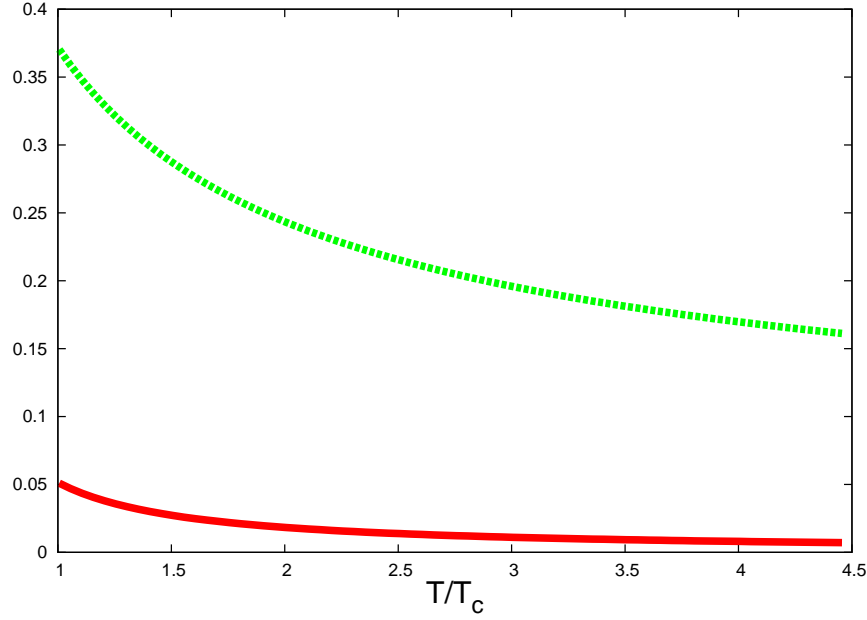


Figure 3: The ratios of the 1st and the 3rd terms on the right-hand side of Eq. (23) to the 2nd term are represented by the lower and the upper curves, respectively.

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